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OF A LASER BEAM REFLECTED FROM A LOCATION TARGET
AND PROPAGATING THROUGH TURBULENT ATMOSPHERE

by

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PROBABILITY DENSITY FUNCTION OF THE INTENSITY OF A LASER BEAM REFLECTED FROM A LOCATION TARGET AND PROPAGATING THROUGH TURBULENT ATMOSPHERE*

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Abstract: From an analysis of the effect of a rough-surfaced target and the effect of atmospheric turbulence on a propagating laser beam, and the double-stochastic field model, we propose an intensity probability density function and the photoelectronic count statistical relationship of a laser beam reflected from a location target and propagating through a weak fluctuating turbulent atmosphere.

1. Introduction

When a laser beam, reflected from a location target, propagates through a turbulent atmosphere, its power, phase and polarization begin varying randomly, due to stochastic fluctuations in the atmospheric refractivity ratio and stochastic fluctuations of scattering surfaces. As a result, this process leads to several effects such as beam expansion, flickering, reduced timewise and spatial coherence, and whole-beam vibrations; these affect the capabilities of laser radar, namely: accurate target positioning, image resolution, and so on, and

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thus have attracted growing interest [1-3]. We discussed the phase-fluctuation statistical distribution [1] of backscattered waves reflected from a surface, given the condition that the rough surface is considered as a simple approximation of turbulent phase screening; additionally, we analyzed the statistical distribution law [4] of the backscattered-wave optical field, given the condition that the spatial nonuniformity of backscattered wave is taken into account. When we take into account, respectively, laser beam modulation by atmospheric turbulence and by the location target, this paper gives a theoretical analysis of the intensity-fluctuation statistical distribution law of backscattered waves reflected by a location target as a laser beam propagates through the turbulent atmosphere, allowing for laser beam modulation by atmospheric turbulence and by location target, respectively.

2. Intensity Probability Density Function of Circular Gaussian Scattering Spots

When a laser beam propagating through a turbulent atmosphere is reflected by a location target surface, it will be subjected to interference owing to atmospheric turbulence and surface fluctuations. Since the target-surface fluctuation law and the atmospheric-turbulence stochastic fluctuation law are statistically independent, we calculate their interference on the laser beam separately. During the analysis, we regard atmospheric turbulence prior to and after scattering by the target as the same interference factor and take the effect of target stochastic fluctuation surface on scattering waves, as another.

Below we give an analysis of the statistical law of laser-beam backscattered waves from a location target surface that is made up of an optically rough part and an optically smooth part. As an approximation, the analysis is limited to surface

scattering in which the backscattered wave spots satisfy the circular gaussian distribution without considering shadowing effects due to the very long distance between observation surface and scattering surface. The backscattered wave total instantaneous field E-arrow is a complex quantity consisting of backscattered waves reflected by the rough-part of the target surface and by the smooth part of the target surface, namely,

$$\vec{E} = \vec{E}_d + \vec{E}_g = E\exp(j\theta) \tag{1}$$

where E and θ are, respectively, the amplitude and the phase of $\vec{E}$; $\vec{E}_d$ and $\vec{E}_g$, respectively, are the scattering stochastic component of the rough part and the backscattered-wave radial component from the smooth part.

Let the stochastic scattered component $\vec{E}_d$ in $\vec{E}$ be a compound gaussian stochastic quantity of zero mean value within the observation surface; the real $E^r{}_d$ and the imaginary $E^i{}_d$ part of $\vec{E}_d$ are gaussian variables, mutually independent and evenly distributed in space while the radial component $\vec{E}_g$ consists of N backscattered waves from the optically smooth part, then

$$\vec{E}_g = \sum_{n=1}^{N} A_n \exp(j\varphi_n) = E_g \exp(j\varphi) \tag{2}$$

where A_n and phi_n are the fixed constant-amplitude and phase of the n-th reflected laser beam from the smooth part, while E_g, phi are the amplitude and phase of the spatial stochastic distribution.

According to [5], the light intensity of field $\vec{E}$ obeys the Rice distribution

$$p_0(I) = \frac{1}{\langle I_d \rangle}\exp\left(-\frac{I + I_g}{\langle I_d \rangle}\right) I_0\left(\frac{2\sqrt{II_g}}{\langle I_d \rangle}\right) \tag{3}$$

where $I = \vec{E}^2, I_g = \vec{E}_g{}^2, \langle I_d \rangle = 2\sigma_d{}^2, \sigma_d{}^2$ are the variance of stochastic scattering component E_d, $I_0(\cdot)$ is the Bessel function of first category zero-order deformation.

Apart from the foregoing description of beam scattering by a location target, the effect of atmospheric turbulence on a propagating laser beam should also be calculated. To simplify analysis, let us assume: 1) the beam propagates only in a weakly turbulent atmosphere, during which modulation of the time-averaged intensity value by beam vibration can be ignored; 2) the linearity of beam emitted to the target is smaller than target linearity; and 3) the spatial distribution of target fluctuation is even. According to the foregoing approximation, Ed is a gaussian variable of zero mean value, and it is assumed to be known that the effect of atmospheric turbulence on backscattered wave from the target surface is virtually an effect on the reflection component in the reflector direction E_g, in other words, E_g is assumed to be a time-varying stochastic variable modulated by atmospheric turbulence, and Eq. (3) is the conditional Rice distribution, then

$$p_0(I/I_g) = \frac{1}{\langle I_d \rangle}\exp\left(-\frac{I+I_g}{\langle I_d \rangle}\right)I_0\left(\frac{2\sqrt{II_g}}{\langle I_d \rangle}\right) \tag{4}$$

While the unconditional probability density function of the backward wave intensity is

$$p(I) = \int_0^\infty p_0(I/I_g)p_t(I_g)\mathrm{d}I_g \tag{5}$$

where $p_t(I_g)$ is the I_g fluctuation probability density function under interference by atmospheric turbulence.

On the assumption of a split scattering model, the light wave radiation field at a given point and at a given instant within the observation surface can be considered as being formed through the superimposition of scattering along different turbulent paths, while the fluctuation distribution of total scattering field intensity x can be expressed with the deformed Rice-Nakagami distribution [6] as follows:

$$p_s(x) = \frac{1}{b}\exp\left[-\frac{(A^2+x)}{b}\right] I_0\left(\frac{2A\sqrt{x}}{b}\right) \tag{6}$$

where A is nonscattered coherent constant-amplitude component and b is the mean wave intensity of the circular compound gaussian stochastic component in the relation as $\langle x \rangle = A^2 + b, \sigma_s^2 = b^2 + 2A^2 b$.

According to N. S. R. Gudimetla et al. [7], the M-distribution is a desired approximation for the deformed Rice-Nakagami distribution. Thus, the M-distribution can be used to approximate the Rice-Nakagami distribution as follows:

$$p_s(x) = \frac{M^M x^{M-1}\exp(-Mx/\langle x \rangle)}{\Gamma(x)\langle x \rangle^M} \tag{7}$$

In this way, the fluctuation distribution of target radial reflection component intensity I_g can be derived as follows:

$$p_s(I_g) = \frac{M^M I_g^{M-1}\exp(-MI_g/\langle I_g \rangle)}{\Gamma(M)\langle I_g \rangle^M} \tag{8}$$

The probability density function of the backscattered wave intensity thus derived is:

$$\begin{aligned}
p_1(I) &= \frac{M^M\exp(-I/\langle I_d \rangle)}{\Gamma(M)\langle I_d \rangle\langle I_g \rangle^M}\int_0^\infty I_g^{M-1}\exp\left[-\left(\frac{1}{\langle I_d \rangle} + \frac{M}{\langle I_g \rangle}\right)I_g\right] I_0\left[\frac{2\sqrt{II_g}}{\langle I_d \rangle}\right]\mathrm{d}I_g \\
&= \frac{M^M}{\sqrt{I\langle I_g \rangle}}\left(\frac{\langle I_d \rangle}{\langle I_g \rangle + M\langle I_d \rangle}\right)^{M-1/2}\exp\left[-\frac{I}{2\langle I_d \rangle}\left(\frac{2M\langle I_d \rangle + \langle I_g \rangle}{M\langle I_d \rangle + \langle I_g \rangle}\right)\right] \\
&\quad\times M_{\frac{1}{2}-M,0}\left[\frac{I\langle I_g \rangle}{\langle I_d \rangle(\langle I_g \rangle + M\langle I_d \rangle)}\right]
\end{aligned} \tag{9}$$

where $M_{k,\mu}(x)$ is the Whittake function and $\Gamma(x)$ is the gamma function.

The nth factorial moment of the backward wave intensity is:

$$
\langle I^n \rangle = \frac{M^M}{\sqrt{\langle I_g \rangle}} \left(\frac{\langle I_t \rangle}{\langle I_g \rangle + M \langle I_t \rangle} \right)^{M-1/2} \int_0^{\infty} I^{n-1/2} \exp\left[-\frac{I}{2\langle I_t \rangle} \left(\frac{2M\langle I_t \rangle + \langle I_g \rangle}{M\langle I_t \rangle + \langle I_g \rangle} \right) \right]
$$

$$
\times M_{\frac{1}{2}-M,0} \left[\frac{I\langle I_g \rangle}{\langle I_t \rangle (\langle I_g \rangle + M\langle I_t \rangle)} \right] dI
$$

$$
= \left(\frac{M\langle I_t \rangle}{\langle I_g \rangle + M\langle I_t \rangle} \right)^M \langle I_t \rangle^n \Gamma(n+1)\, {}_2F_1 \left[n+1, M, 1, \frac{\langle I_g \rangle}{\langle I_g \rangle + M\langle I_t \rangle} \right] \tag{10}
$$

where ${}_2F_1(a, b, c, x)$ is the hypergeometric function.

If the location target is much smaller than transmitting beam, then, taking advantage of the statistical independence of the atmospheric refractivity ratio fluctuations and the target surface fluctuations, light wave scattering by target can be taken as re-modulation of the radial component of scattered wave in atmospheric turbulence. Similarly, by writing. Eq. (6) as $p_t=p_i(x/A^2)$ and by writing Eq. (4) as $P_0(A^2)$ in the M-distribution approximation, the intensity probability density function of backscattered-wave field can be obtained as follows:

$$
p_2(I) = M^M \left(\frac{1}{\langle I_t \rangle + M\langle I_g \rangle} \right)^M \langle I_g \rangle^{M-1} \exp\left(-\frac{I}{\langle I_g \rangle} \right) {}_1F_1 \left[M, 1, \frac{I\langle I_t \rangle}{\langle I_g \rangle (\langle I_t \rangle + M\langle I_g \rangle)} \right] \tag{11}
$$

where ${}_1F_1(a,b,x)$ is the confluent hypergeometric function.

Individual factorial moments of light intensity corresponding to Eq. (11) are:

$$
\langle I^n \rangle = \left(\frac{M\langle I_g \rangle}{\langle I_t \rangle + M\langle I_g \rangle} \right)^M \langle I_g \rangle^n \Gamma(n+1)\, {}_2F_1 \left[M, n+1, 1, \frac{\langle I_t \rangle}{\langle I_t \rangle + M\langle I_g \rangle} \right] \tag{12}
$$

Since our analysis focuses on the intensity fluctuation of backscattered wave from a location target during propagation through a stable atmosphere with turbulence from gentle to weak, the integral intensity corresponding to Eq. (9) has the same distribution as instantaneous intensity [8], W= $\int_t^{t+\tau} I dt$, that is,

$$
p_1(W) = \frac{M^M}{\langle W_g \rangle} \left(\frac{\langle W_t \rangle}{\langle W_g \rangle + M\langle W_t \rangle} \right)^M \exp\left[-\frac{W}{\langle W_t \rangle} \right] {}_1F_1 \left[M, 1, \frac{W\langle W_g \rangle}{\langle W_t \rangle (\langle W_g \rangle + M\langle W_t \rangle)} \right],
$$

According to Mandel's equation, the optoelectronic count

distribution is

$$p_1(m,T) = \int_0^\infty \frac{(aW)^m}{m!} e^{-aW} p_1(W)\,dW$$

$$= \left(\frac{M\overline{m}_t}{\overline{m}_g + M\overline{m}_t}\right)^M \frac{\Gamma(m+1)}{m!\,\overline{m}_g} \left(\frac{\overline{m}_t}{1+\overline{m}_t}\right)^{m+1}$$

$$\times {}_2F_1\left[M, m+1, 1, \frac{\overline{m}_g}{(\overline{m}_g + M\overline{m}_t)(1+\overline{m}_t)}\right]$$

(13)

where a is the ratio factor, $\overline{m}_t = a\langle W_t\rangle, \overline{m}_g = a\langle W_g\rangle$.
In the same way, $p_2(m,T)$, which corresponds to Eq. (11), can be derived.

3. Intensity Probability Density Function of Elliptic Gaussian Scattering Spots

Generally speaking, the compound amplitude of scattered spots from a rough surface can satisfy the elliptic gaussian distribution [9] and therefore, it is necessary to analyze the intensity probability density function of backscattered waves reflected from by this target during beam propagation through a turbulent atmosphere. Since we can somehow simplify, through coordinate conversion, the general elliptic gaussian distribution into the product of distribution functions corresponding to independent components, the combined probability density function of the real part and the imaginary part of scattered spot amplitude can be expressed as:

$$p(E_z, E_y) = \frac{1}{2\pi\sigma_z\sigma_y} \exp\left[-\frac{(\Delta E_z)^2}{2\sigma_z^2} - \frac{(\Delta E_y)^2}{2\sigma_y^2}\right] = p(E_z)p(E_y) \tag{14}$$

where E_z and E_y, respectively, are the real part and the imaginary part of $\vec{E}$, $\Delta E_z = E_z - \langle E_z\rangle$ is the fluctuation quantity of E_z, $\langle E_z\rangle = N\exp(-\sigma^2/2)$ is mean value of the E_z component, $\sigma_z^2 = \langle \Delta E_z^2\rangle = (N/2)[1 + \exp(-2\sigma^2) - 2\exp(-\sigma^2)]$ is fluctuation

variance of E_z, $\sigma_y^2 = \langle E_y^2 \rangle = (N/2)[1-\exp(-2\sigma^2)]$ is fluctuation variance of E_y, <·> is the system statistical average, N is the number of target scattering elements, σ is the phase fluctuation standard difference of target scattering field. By making an analysis similar to that described in reference [4], we acquire:

$$p(I,\theta) = \frac{1}{2\pi\varphi\langle I_t\rangle}\left\{\exp\left[-\frac{2(I+I_r-2\sqrt{II_r}\cos(\theta-\mu))}{(1+\varphi)\langle I_t\rangle}\right] - \exp\left[-\frac{2(I+I_r-2\sqrt{II_r}\cos(\theta-\mu))}{(1-\varphi)\langle I_t\rangle}\right]\right\} \tag{15}$$

By using relations between joint probability density and marginal probability density, the monovalent probability density function of light intensity can be derived as follows:

$$p(I) = \frac{1}{\varphi\langle I_t\rangle}\left\{\exp\left[-\frac{2(I+I_r)}{(1+\varphi)\langle I_t\rangle}\right] I_0\left[\frac{4\sqrt{II_r}}{(1+\varphi)\langle I_t\rangle}\right] - \exp\left[-\frac{2(I+I_r)}{(1-\varphi)\langle I_t\rangle}\right] I_0\left[\frac{4\sqrt{II_r}}{(1-\varphi)\langle I_t\rangle}\right]\right\} \tag{16}$$

Upon consideration of I_g modulation by atmospheric turbulence, when the target is larger than beam linearity, the intensity probability density function of backscattered waves by a location target in turbulent atmosphere propagation can be derived from Eq. (5) as follows:

$$\begin{aligned} p_3(I) = {} & \frac{1+\varphi}{2\varphi}\left(\frac{M(1+\varphi)\langle I_t\rangle}{\langle I_r\rangle + M(1+\varphi)\langle I_t\rangle}\right)^M \exp\left[-\frac{2I}{(1+\varphi)\langle I_t\rangle}\right] \\ & \times {}_1F_1\left[M,1,\frac{I\langle I_r\rangle}{(1+\varphi)\langle I_t\rangle(\langle I_r\rangle + M(1+\varphi)\langle I_t\rangle)}\right] \\ & - \frac{1-\varphi}{2\varphi}\left[\frac{M(1-\varphi)\langle I_t\rangle}{\langle I_r\rangle + M(1-\varphi)\langle I_t\rangle}\right]^M \exp\left[-\frac{2I}{(1-\varphi)\langle I_t\rangle}\right] \\ & \times {}_1F_1\left[M,1,\frac{I\langle I_r\rangle}{(1-\varphi)\langle I_t\rangle(\langle I_r\rangle + M(1-\varphi)\langle I_t\rangle)}\right] \end{aligned} \tag{17}$$

Correspondingly, the various intensity factorial moments are

$$\begin{aligned} \langle I^n\rangle = {} & \frac{1+\varphi}{2\varphi}\left[\frac{M(1+\varphi)\langle I_t\rangle}{\langle I_r\rangle + M(1+\varphi)\langle I_t\rangle}\right]^M [(1+\varphi)\langle I_t\rangle]^n \Gamma(n+1) \\ & \times {}_2F_1\left[n+1,M,1,\frac{\langle I_r\rangle}{\langle I_r\rangle + (1+\varphi)\langle I_t\rangle}\right] \end{aligned}$$

$$
-\frac{1-\varphi}{2\varphi}\left[\frac{M(1-\varphi)\langle I_t\rangle}{\langle I_s\rangle + M(1-\varphi)\langle I_t\rangle}\right]^M \left[(1-\varphi)\langle I_t\rangle\right]^n
$$

$$
\times \Gamma(n+1)_2F_1\left[n+1, M, 1, \frac{\langle I_s\rangle}{\langle I_s\rangle + (1-\varphi)\langle I_t\rangle}\right] \tag{18}
$$

Similar methods can also be used to obtain corresponding results when the location target is smaller than beam linearity.

4. Conclusion and Discussion

Based on the statistical independence of atmospheric turbulence fluctuations and target surface fluctuations, as well as their separate modulation capability over a propagating laser beam, this paper has derived the flicker probability density function of backscattered waves reflected by a rough surface during propagation through turbulent atmosphere as expressed in Eqs. (9), (11) and (17). These results indicate that intensity fluctuation distribution under joint interference by atmospheric turbulence and location target proves to differ from the propagating laser intensity fluctuation distribution under the single-existence of either stochastic rough surface or atmospheric turbulence. This deserves attention when strict calculations are conducted over the intensity density function of backscattered waves reflected from a target during propagation through turbulent atmosphere.

Our analysis shows that beam vibration affects the measurement of average light intensity and therefore affects the light flicker probability distribution [6]; also, using the M-distribution as replacement of the approximate Rice-Nakagami distribution, given the condition that the spherical wave source and parameter M>5 appears to be a desired approximation with an error<3%. However, in our analysis, the beam vibration effect is ignored, and the M-distribution is adopted to approximate Rice-Nakagami distribution, the results thus obtained are applicable only to flickering probability distribution of the

reflected beam from a rough surface during divergent spherical wave propagation through a weakly turbulent fluctuation region.

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